Theorem 0.1. *For any set A, we have $A \lneq_T A'$, where $A' = \{x : \phi_x^A(x)\downarrow\}$.*

Proof. There are two parts to this proof: $A \leq_T A'$, and $A' \not\leq_T A$.
We first show $A \leq_T A'$. Given access to the oracle for A', we wish to construct a Turing machine M that decides whether some arbitrary $x \in \mathbb{N}$ is in A or not. Consider the following procedure:

$M(x)$: Construct (but don't run) the following A-oracle Turing machine M_x:
  $M_x(y)$: (Ignore y)
    If $x \in A$:
      Halt.
    Else:
      Loop.
  Find the Gödel number e_x of M_x, so that $M_x(y) = \phi_{e_x}^A(y)$.
  If $e_x \in A'$:
    Accept.
  Else:
    Reject.

Notice that $M(x)$ constructs an A-oracle Turing machine M_x, but never actually consults the A-oracle as we don't run M_x. Also notice that $M_x(y)$ halts if and only if $x \in A$, regardless of the value of y.
So $x \in A$ if and only if $M_x(e_x)$ halts, which happens if and only if $\phi_{e_x}^A(e_x)\downarrow$. So $M(x)$ accepts if and only if $x \in A$, which means M (which uses an A'-oracle) serves as a decider for A. Thus $A \leq_T A'$.
To show $A' \not\leq_T A$, suppose towards a contradiction that $A' \leq_T A$. So $I_{A'}$ is A-computable. Define the following function f:

$$f(x) = \begin{cases} 0 & I_{A'}(x) = 0 \\ \text{undefined} & I_{A'}(x) = 1. \end{cases}$$

If we can compute $I_{A'}$ then we can compute f, and since $I_{A'}$ is A-computable, this means f is A-partial computable. So $f = \phi_e^A$ for some $e \in \mathbb{N}$.
If $e \in A'$, then $I_{A'}(e) = 1$, so $f(e)$ is undefined. But $e \in A' \Leftrightarrow \phi_e^A(e)\downarrow$, and since $f = \phi_e^A$, $f(e)$ should be defined, contrary to the previous sentence.
If $e \notin A'$, then $I_{A'}(e) = 0$, so $f(e)$ is defined. But $e \notin A' \Leftrightarrow \phi_e^A(e)\uparrow$, so $f(e)$ should be undefined, again a contradiction.
We conclude that $A' \leq_T A$ is false.